Estimation of Lesotho's Yield Curve

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Abstract

This paper builds on previous work by the Central Bank of Lesotho to estimate Lesotho's yield curve. Its purpose is to contribute to capital market development goals by helping encourage corporate bond issuances, improving pricing of financial products, and providing guidance as to yields that will be achieved at government bond auctions. Since South African zero-coupon yields are available on a daily basis, the model developed in this paper harnesses this information to estimate Lesotho's zero-coupon yield curve at any date. The model performs well as measured by both in-sample and out-of-sample testing, producing negligible bias.

JEL classification: G12, E43, E47

Key words: Zero-coupon yields, Government bonds, Yield curve

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1. INTRODUCTION

Trading of government or default-free securities produces important financial information useful to public, private, financial and non-financial entities alike. In particular, “risk-free” rates of return, normally calculable from bond or swap prices, contain important information about macro-economic conditions, market expectations of future real interest rates and inflation, as well as the time value of money. They can be used for many purposes, including the pricing of insurance products, as a benchmark for bank lending, the pricing of other financial or investment products such as derivatives, the valuation of pension liabilities, as an input into the benchmarking of a project’s Internal Rate of Return (IRR), and general assessment of an economy’s prospects.

In Lesotho, financial markets are underdeveloped and thus highly illiquid. While treasury bills are frequently issued and some medium- and long-term treasury bonds are occasionally issued and re-opened, there is little to no observable secondary market trading in either. This means that Lesotho’s government bond interest rates are normally not known with precision, especially for longer maturities. As a result, commercial and government entities in Lesotho therefore miss out on the benefits of the information normally provided by government bond yields when making pricing and capital allocation decisions.

The purpose of this paper is therefore to use available information in order to build a model to estimate, at any date, Lesotho government’s zero-coupon yield (ZCY) curve (the plot of government interest rates for different maturities of zero-coupon bonds). Of particular interest are maturities of greater than one year, given that ZCYs of maturities up to one year are generally known due the aforementioned regular and frequent issuance of treasury bills. Of course such an estimate will not provide the same level of information as would exist if Lesotho’s financial market were more fully developed. Further, there may arguably be a non-negligible chance of the Lesotho government defaulting on its obligations, meaning that such a ZCY curve would not be risk-free. However, the paper is prepared on the basis that such an estimate is nevertheless valuable, as long as the uncertainty of the estimate is well understood.

The model developed is expected to contribute to capital market development goals through the information it provides. It is expected to help encourage corporate bond issuances, improve private sector ability to price financial products such as insurance and loans, and provide guidance regarding yields that will be achieved on issuance or re-opening of government bonds. As such it should contribute to the improvement of overall capital allocation in Lesotho.

Other benefits from this project and the model developed include the following.

- Help establish the Central Bank of Lesotho's (CBL) reputation as a provider of timely and useful financial market information in Lesotho
- Potentially (marginally) reduce the government's cost of capital, to the extent that better information leads to less uncertainty surrounding Lesotho government securities and thus greater demand for them
- Attract (marginal) additional capital from abroad since investors will better understand local market conditions
- Draw additional traffic to CBL's website

The remainder of this paper is structured as follows. Section 2 summarises previous work by CBL on estimating Lesotho's yield curve. Section 3 briefly touches upon relevant literature not covered by the aforementioned CBL work. Section 4 describes the methodology adopted and its specific rationale given Lesotho's circumstances. The data used, including the process used to establish historic data points for Lesotho's ZCYs is also captured in section 4. Section 4 further explains the preferred model to estimate Lesotho's ZCY curve based on that of South Africa (SA) at any date, and sets out the estimation results for its parameters. Section 5 provides testing results for bond yield estimates produced, including the preferred model and a number of other models that were also considered. Section 6 concludes and Section 7 provides recommendations to take the project forward into implementation..

2. PREVIOUS WORK BY THE CENTRAL BANK OF LESOTHO

Earlier work on estimating Lesotho's ZCY curve culminated in two main papers. The first was 'Modelling Yield Curve in Lesotho' by Molise *et al.* (2010). In this paper, the authors advocated for the adoption of the bootstrapping method in order to measure Lesotho's yield curve. The bootstrapping method was chosen for its balance between simplicity, ease of computation, accuracy and purpose. For the purpose of extrapolation to maturities beyond five years (the longest-dated bond on issuance in 2010), a "shape factor" assumption was made.

The second was a paper entitled 'Lesotho Yield Curve – Simple Parametric Model (Nelson-Siegel)' by Noosi B. and Nthontho T. (2014). The authors of this paper adopted a Nelson-Siegel model, one type of parametric model that is commonly-used for modelling the yield curve. The model was selected based on its good prediction ability, its wide use among many Central Banks and financial markets practitioners, its simplicity compared to other parametric models, and its flexibility. In the paper, the authors fit a Nelson-Siegel model to treasury bill data (up to a term of 1 year) and use it to extrapolate zero-coupon yields up to a term of 2 years.

3. LITERATURE REVIEW

Due to the relatively comprehensive review of yield curve modelling techniques in previous CBL work (e.g. Molise *et al*. 2010), this section does not replicate this work. Rather, it seeks to focus on additional literature and peer benchmarking not covered by previous work.

Many techniques are available to address the issue of yield curve smoothing, based on knowledge of some points on the ZCY curve. One of the most commonly used is the Nelson-Siegel model, which can be used to fit a curve to the available interest rate data points at a particular date. This model is described (alongside others) by CBL in Molise *et al*. (2010) and used in Noosi and Nthontho (2014). Some papers use these techniques in the context of secondary markets for government securities with low trading volumes (Chakroun and Abid 2013, Chou *et al*. 2009, Vaidyanathan *et al*. 2002). However, given the lack of existence of a secondary market in Lesotho, these techniques are not as readily applicable. For example, it is only possible to use a Nelson-Siegel curve to estimate longer-term maturities on days when there is trading data for longer-term securities. In Lesotho, this is only the case when bonds are issued or re-opened.

However, the peg of the Loti to the Rand creates an opportunity to consider the use of SA financial market information. As explained in Box 1, another country in a similar situation to Lesotho in this regard is Denmark. That is, the Danish Krone is pegged to the Euro, and Denmark's bond and swap markets are less liquid relative to those of some countries in Europe, e.g. Germany (albeit much more liquid than Lesotho's markets). Danish and European authorities therefore use the euro swap yield curve with an adjustment, for the purpose of valuing Danish insurance liabilities (Danish FSA 2004, CFO Forum and CRO Forum 2010).

Box 1: Danish yield curve for insurance liability valuation

In deriving a yield curve, Denmark faces a similar problem to Lesotho due to the small, relatively illiquid nature of its government bond market. Combined with the fact that its currency (the krone) is pegged to the Euro, this means that Denmark's situation bears important similarities to that of Lesotho. In 2004, the Danish financial regulator, Finanstilsynet (Danish FSA), sought to develop a method for deriving Denmark's yield curve for the purpose of insurance company regulation (Danish FSA, 2004). Specifically, in order to require insurance companies to report their liabilities' market value, a discount rate for each liability cash flow had to be specified. The previous regulation regime, which allowed a single discount rate to be applied to all liabilities regardless of term, was considered inadequate.

In specifying a yield curve, the use of government bonds was viewed negatively due to the scarcity of long-term Danish government bonds (i.e. maturity of greater than 10-12 years)—only one such bond existed. Further, liquidity in the government bond market was a problem, particularly for long term yields, meaning that yields are prone to influence by individual large transactions. Lastly, there was little indication that long-term government bond issuance would be sustained in future.

Danish FSA also considered drawing upon the domestic interest rate swap market, which are effectively the interest rates at which banks lend to other banks. While this option is preferable to the use of government bonds, the swap market is still illiquid relative to euro swaps, and similarly to government bonds it is prone to influence by individual large transactions due to Denmark's small market.

Therefore, the option that Danish FSA chose was to use the euro swap yield curve, with an adjustment for the credit risk of Denmark relative to Euro countries. The reason for this choice was that Danish FSA decided to place significant weight on market liquidity, in order to achieve smoothness and stability of the yield curve to be used. The benefits of the much larger and more liquid euro swap market were thus considered sufficient to outweigh the disadvantage of not closely reflecting any changes in the relationship between the Danish and the Euro yield curve, as could have been better achieved through use of the Danish swap rates. This disadvantage is nevertheless minimized through the adjustment for credit risk, which is based on the observed difference between Danish and German government yields (Danish FSA, 2011).

Applying the Danish experience to Lesotho, it is interesting to note the three problems identified with the use of the government bond market to derive a yield curve: scarcity of long-term bonds, illiquidity, and uncertain future government issuance. Since these problems also apply to Lesotho's government bond market but to a much more severe degree, this would argue against relying only on Lesotho government bonds for the purposes of this paper. Thus, since Lesotho does not have an observable domestic swap market, this leaves the option of using the SA interest rate swap curve with adjustment. Accordingly, the remainder of this paper proceeds to develop a method of predicting Lesotho yields based on SA yields.

4. METHODOLOGY

As explained above, approaches taken to date have consisted of applying smoothing methods (bootstrapping in once instance, and the Nelson-Siegel model in the other). Both of these approaches rely on the availability of longer-term yield observations on the same date for which the yield curve must be estimated, which is an unfortunately restrictive condition in Lesotho.

This restriction can be relaxed by making use of the abundant information produced by SA's financial markets. The high degree of reliability of the peg of Lesotho's Loti to SA's Rand means that there is a strong connection between the two countries' interest rates, since any investor can readily choose between investing in SA's vs. Lesotho's money market without exchange rate fluctuations being a concern. In fact, under simplified circumstances, Lesotho's yield curve would actually be identical to SA's. In practice, Lesotho government bonds tend to exhibit higher yields than SA government bonds. Nevertheless, any approach that ignores this relationship is likely to produce suboptimal estimates of Lesotho's yield curve.

Therefore, this paper seeks to model the Lesotho zero-coupon yield curve based on that of SA, using appropriate adjustments. Specifically, the approach taken is to use the bond yield data available from issuances and re-openings of Lesotho treasury bonds and bills in order to estimate Lesotho's Zero-Coupon Yield (ZCY) curves at particular dates when data is sufficient for this. A generalised bootstrapping method is used for this. The resulting ZCYs are then regressed on their SA equivalents in order to develop a model for estimating the Lesotho ZCY curve—a variety of regression models are considered, as set out later in this paper. This model can produce estimates for the Lesotho ZCY curve on any day, since data for SA's ZCY curve are available on a daily basis.

4.1 Data and Bootstrapping Methodology

(a) Data

For this study, information was obtained from CBL's Financial Markets Department (FMD) on the yields obtained at issuances and re-openings of treasury bonds and bills, as well as on the characteristics of the bonds and bills in question. As mentioned earlier, data for Lesotho's treasury bills are more plentiful than for its treasury bonds. Nevertheless, there have been 36 issuances/re-openings of treasury bonds on 19 dates since the commencement of this market in 2010, which is sufficient for achieving useful results in this study.

The dynamics of these auctions are that the government typically tries to issue or re-open bonds at par, or at a premium. So far it has succeeded; out of the 36 issuances/re-openings, 29 have been at par and 7 have been at a premium. Although this behaviour may influence the yield obtained, ultimately bond yields anywhere are driven by both supply and demand, with the supply of government bonds being determined by the government's issuance preferences. Therefore, these transactions still provide relevant information about Lesotho's yield curve.

Data used for the SA yield curve is based on zero-coupon swap mid rates, which are available from Bloomberg on a daily basis (ICVS Curve ID: ZAR.3M, Field: ZC.MID). Although there may be small differences between swap yields and bond yields, either is sufficient for the purpose of estimating the Lesotho yield curve, as any differences are likely to be accounted for by the regression model.

(b) Bootstrapping Methodology

The Lesotho bond yields in the data from CBL's FMD are bi-annually compounded bond yields, and thus cannot be directly compared to the SA ZCY curve. Therefore it is necessary to derive Lesotho's ZCY curve on each auction date from the bond yields observed. Although the ZCYs are observable for terms up to and including one year due to the Treasury Bill auctions, beyond the short term tenors a bootstrapping exercise is required for each bond auction date to estimate the ZCY curve for that date. Historically in Lesotho, two bonds have normally been sold on each auction date (with two exceptions where only one bond was sold), leading to two data points at those dates where this has been the case. Combining this with the nearest Treasury Bill auction data (typically from a week earlier), this leads to six yield curve data points for each bond auction date. This data is included in Appendix I. For each auction date, the bootstrapping methodology is therefore applied to these six data points to obtain the ZCY curve for that date.

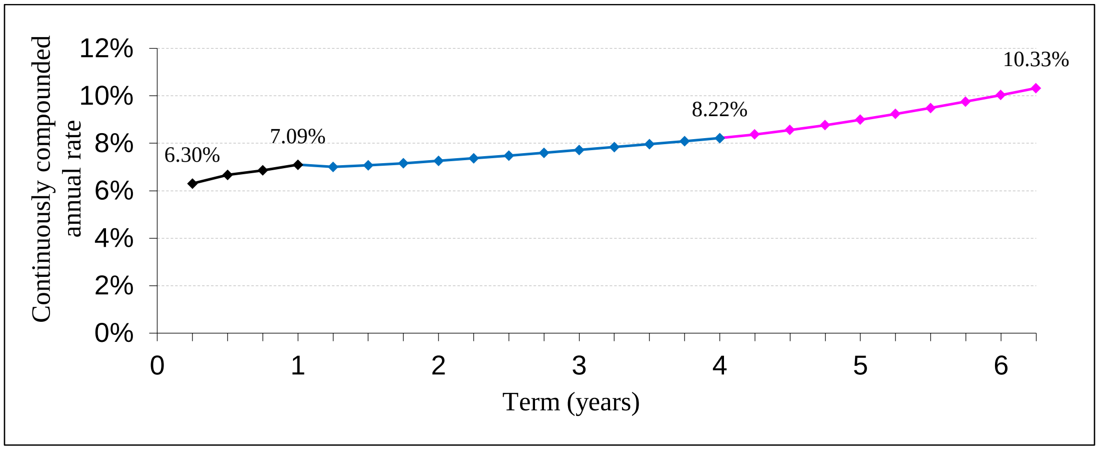
Although this process produces a full ZCY curve for each date, it is unlikely that all ZCYs produced are reliable given the relatively few data points beyond the payment term of 1 year. Therefore, since the final bond payments are by far the largest and thus have much more weight in the calculations than the bonds' coupon payments, the most reliable ZCYs are those that coincide with the bond's maturities. Therefore, only those ZCYs from the bootstrapped yield curve are collected for analysis.

In summary, this method produces one ZCY for each treasury bond at each auction, for a total of 36 observations. This data is complemented with yield data from the (zero-coupon) treasury bills that are issued around the same time as the treasury bonds, leading to 76 observations for treasury bills and a total dataset of 112 observations. It would be possible to incorporate more treasury bill data from other auction dates into the analysis. However, since the main focus of this study is to estimate longer-term ZCYs, this was not done, for fear of overshadowing the longer-term data points in the analysis.

To calculate the ZCY curves, two bootstrapping methodologies are used. The first is based on the Nelder-Mead numerical method for non-linear optimisation of an objective function in multidimensional space. It is capable of calculating a zero-coupon yield curve that produces the correct bond prices given the coupon and maturity payments of the two bonds and the four treasury bills at a given date. The Nelson-Mead method produces smooth yield curves as per Figure 1. Given a limitation of the software used to implement the Nelder-Mead method, bond payment terms are rounded to the nearest quarter for both methods.

Figure 1: Estimate of Lesotho Zero-Coupon Yield Curve

Nelder-Mead Method, 18 March 2015

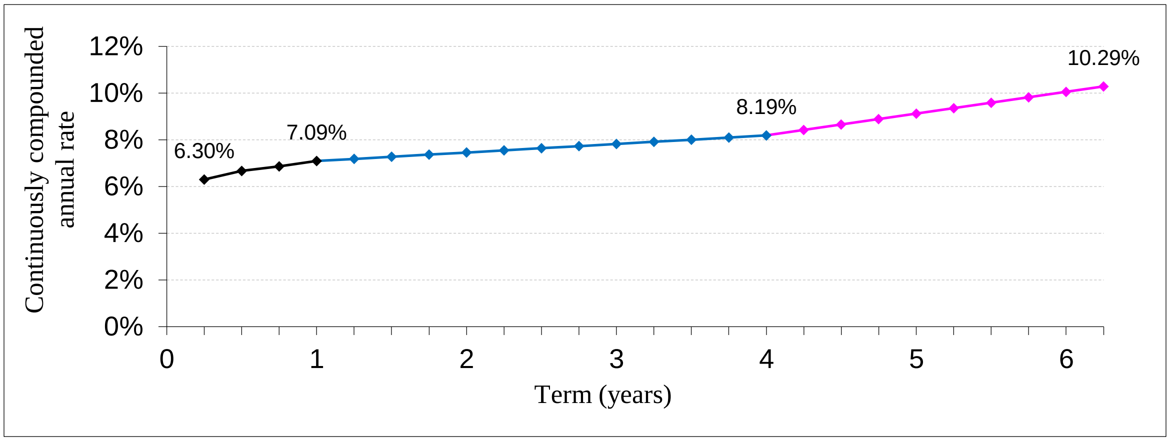


Source: CBL

The second method used is that of a linear spline, as illustrated in Figure 2 for 18 March 2015, when the 7 and 10 year bonds (maturing in 4 and 6.25 years respectively as of that date) were re-opened. Starting from the 1-year Treasury Bill yield, a constant amount of ZCY increase per year of payment term is assumed, which is chosen such that the resulting ZCYs value the 7-year bond's remaining payments accurately. For terms that are later than 4 years (i.e. after the maturing of the 7-year bond), a different constant amount of ZCY increase per year of payment term is assumed, which is chosen such that the resulting ZCYs value the 10-year bond's remaining payments accurately. In the example illustrated, the two methods produce ZCYs that are very close, although this is not always the case.

Figure 2: Estimate of Lesotho Zero-Coupon Yield Curve

Linear Spline Method, 18 March 2015



Source: CBL

SA zero-coupon yields corresponding to the yields calculated by the methods above are obtained by referring to the government swap yield curve for the same date, and applying linear interpolation where the exact term cannot be matched. For example, in Figure 1, the Nelder-Mead method produces the result that the 6.25 year ZCY in Lesotho was 10.33% on 18 March 2015. The equivalent SA yield would be obtained by taking 75% of the 6-year SA ZCY and 25% of the 7-year SA ZCY on that date.

4.2 Model Formulation and Results

(a) Model Formulation

The model proposed for Lesotho's ZCY curve is one whereby the variation in a Lesotho ZCY of any term is determined by a constant, the corresponding SA ZCY and the natural log of the term in years. This model is not a time-series model; it is assumed that any time-related effects such as serial correlation are adequately captured through the inclusion of the SA ZCY data.

$$L_i = a + b.SA_i + c.ln(term_i) + \epsilon_i \quad (1)$$

Where:

- i = observation index, corresponding to a particular security at a particular date
- L_i = Lesotho's estimated continuously compounded annual zero-coupon yield

- SA_i = SA's continuously compounded zero-coupon swap yield
- $term_i$ = years until the payment is made
- ϵ_i = error term
- a, b and c = parameters to be estimated

The rationale behind this formulation is as follows. We expect Lesotho's ZCY to be closely related to SA's yield. However, we also expect factors such as default risk and illiquidity premia, which affect bond yields (see Longstaff et al. (2005) and Hull et al. (2005) which focus on corporate credit risk), to differ between SA and Lesotho. In terms of default risk, since it is likely that there is an increased level of country risk in Lesotho relative to SA, this may result in a default risk premium relative to SA bonds. This risk premium might be captured by the constant term, since sovereign default risk affects all government bonds equally in each time period. We also expect there to be an illiquidity premium on Lesotho bonds relative to SA bonds. Illiquidity, as measured by years required to liquidate (sell or redeem) an investment asset, increases in line with payment term in the case of Lesotho government bonds, meaning that the longer the tenor of a bond the greater the premium required each year by investors. This provides a rationale for including $\ln(term_i)$ as an explanatory variable, to capture illiquidity and any other term-varying risk premia. The use of the natural log of the payment term, instead of the payment term without any transformation, helps account for non-linearity of term-dependence, and in practice was found to eliminate the problem of non-normality of the error term.

Impulse and other types of dummy variables have been avoided in the construction of the model, since the authors are not aware of any reasons why particular observations or groups of observations deserve nullification. In particular, one cannot suppose that any observation or group of observations in the dataset is representative of an event that will never re-occur in the future.

5. EMPIRICAL RESULTS

Upon examination of the results presented in Table 1, it is evident that both bootstrapping methodologies produce acceptable results that are in line with prior expectations. That is, the explanatory variables used appear to be relevant and the co-efficient of SA_i (represented in the estimation results as SA_YIELD) is reasonably close to one, consistent with the intuition that a large part of the Lesotho ZCY curve is driven by the SA ZCY curve.

The high R^2 values and the highly significant t-statistics, particularly on SA_i, do imply that some cointegration may be present. This is plausible given the relationship we expect between Lesotho and SA ZCYs. Explicitly modelling any the cointegration relationship, or more generally taking a more sophisticated modelling approach, may be a topic for further research. However, it is far from obvious that such a model would produce more reliable ZCY estimates than the simple ones shown above. Moreover, the possibility of super-consistency in the above regressions does not invalidate the parameter estimates. It only indicates that the standard errors, and thus the t-statistics and p-values for individual coefficients, cannot be relied upon. It is therefore preferable to rely on statistics such as the Schwarz Information Criterion (SIC) in order to select between model specifications.

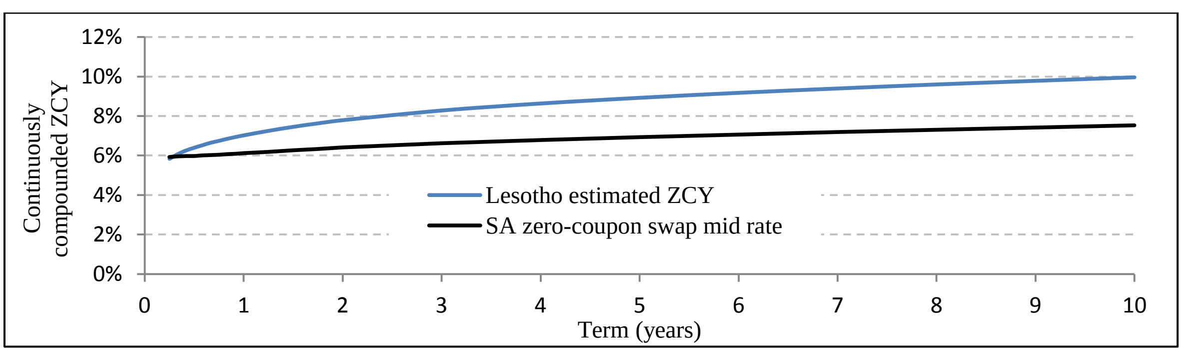
Table 1: Empirical Results

Dependent Variable: *LESOTHO_YIELD*		
	Model I	**Model II**
C	0.0159***	0.0151***
	(3.011)	(2.842)
SA_YIELD	0.8870***	0.9029***
	(9.632)	(9.803)
LOG(TERM)	0.0073***	0.0073***
	(15.009)	(15.206)
Adjusted R^2	0.92	0.92
F-Statistic	652.001	673.623
[P-value]	[0.00]	[0.00]
Wald F-Statistic	451.02	473.21
[P-value]	[0.00]	[0.00]
Jarque-Bera Normality [P-value]	0.591	0.508
Breuch-Pagan-Godfrey Heteroscedasticity [P-value]	0.00	0.00
Number of Observations	112	112

Note: */**/*** denotes significance at 10,5 and 1 per cent levels, respectively. T-ratios are in italics and parenthesis. Model I and Model II, using Nelder-Mead and liner method for bootstrapping, respectively.

Figure 3: Comparison of estimated Lesotho ZCY curve with SA ZCY curve

As at 4 March 2015



Source: CBL, Bloomberg. Lesotho ZCY estimated using regression model calibrated from Nelder-Mead bootstrapped ZCY curve data.

It is also noteworthy that both the above models exhibit heteroskedasticity, primarily because short-term ZCYs are subject to different sources of variation than long-term ZCYs. This again leads to unreliable standard errors, but does not invalidate the estimates of the parameters, meaning that the model can still be used for the desired purpose. Heteroskedasticity can be avoided by omitting the short-term ZCYs (payment terms less than or equal to 1 year) and focusing solely on longer-term ZCYs. However, as explained in Section 7, this leads to other problems.

(a) Relative Importance of Model Diagnostics and Interpretation

It is imperative to bear in mind that this model, in accordance with the purpose of this project, is primarily designed to produce ZCY estimates. Therefore, while statistical diagnostics were used as tools in the development the model, the model is not optimised to have the best SIC or other diagnostic statistics, as good diagnostics can belie poor estimation power. Likewise, while qualitative understanding guided the methods used and variables included, the model is not designed to be the best model for developing understanding of the sources of difference between the SA and Lesotho ZCY. For example, the estimated parameters for the constant and for the coefficient of $ln(term_i)$ are unlikely to correspond exactly to the country risk and illiquidity factors discussed earlier. This is because, apart from these concepts being difficult to quantitatively isolate, the parameters mentioned may be functioning as a substitute for some component of the SA interest rate, particularly since the estimated coefficient of SA_i is less than one. A more informative study into these aspects could be achieved by having as the dependent variable the difference between the Lesotho and SA ZCY, but this is outside the scope of this paper.

Accordingly, the following section focuses on the power of the combined (bootstrapping and regression) modelling approach to predict bond yields. The following section also uses this testing approach to investigate some alternative formulations of the regression model.

(b) Tests of Model Accuracy

It is impossible to test the ZCYs generated by the bootstrapping process since Lesotho's ZCYs are not observable. It is possible to test the ZCYs that are estimated by the regression model against the ZCYs derived from the bootstrapping process, but this is less useful because the latter are themselves only estimates, not actual market data. Therefore, to test the model, this paper takes the approach of using the ZCYs estimated by the regression model to calculate the overall yields of bonds at their points of issuance/re-opening, and comparing these estimated bond yields with those actually observed in the market. It should be noted that such a comparison functions as a test of the entire approach taken in this paper, including both the bootstrapping methodology and the regression model for ZCYs.

Test metrics used include the Mean Absolute Error (MAE), the Root Mean Squared Error (RMSE), and the average bias of bond yield estimates. Testing focuses purely on estimates of treasury bond yields, not treasury bill yields, since the purpose of this paper is to provide insight into longer-term yields (up-to-date information on short-term yields normally exists due to approximately fortnightly issuances of treasury bills for monetary policy purposes). Testing is conducted on both an in-sample and out-of-sample basis. Out of sample testing is conducted in three phases, estimating the first third of bond yield data (using the remaining two thirds of data to estimate the model's parameters), then the second third, then the final third. Differences between estimates and actuals from the three phases are then used to calculate the relevant test metrics. In order to simulate the fact that in reality the model would be used to estimate the yield obtainable on a bond that is not yet issued, SA ZCYs from two weeks earlier than the bond's issue date are used, rather than using the SA ZCYs from the same day as the bond's issuance or re-opening date.

In addition to investigating the relative merits of the Nelder-Mead vs. linear spline method for estimating Lesotho's ZCY curve, a number of different equations for using the SA ZCYs to estimate the Lesotho ZCYs were investigated. The equations tested are set out in Table 3, and the results are presented in Table 4. It should be noted that the out of sample test results hold more weight than the in-sample results, since they represent a more stringent and realistic test. However, it is also important to include qualitative considerations in deciding on a model, since even out-of-sample testing is limited to the range of data experienced to date.

Of the models tested, the equation originally introduced in Section 0 using the Nelder-Mead method is preferred. The model makes intuitive sense, with a SA_i coefficient of close to one and a logical set of regressors that accord with prior expectations. It also performs well based on the above testing metrics, featuring negligible bias and a MAE and RMSE of less than 0.5%, meaning that more often than not we can expect this model's prediction to fall within 0.5% percentage points of the actual bond yield.

A close contender is Equation 1, which offers essentially the same forecasting performance but a SA_i coefficient that is slightly further away from 1. Otherwise, the best alternative candidates for modelling Lesotho's ZCY curve are Equations 4 and 5, when the Nelder-Mead method is used (with the linear spline method, these equations lead to biased estimates based on the above test results). These equations are roughly on par with the preferred equation (and Equation 1) based on the above statistics. However:

- Equation 4's use the natural log of SA_i is less intuitive, given that we think of Lesotho interest rates as being directly related to SA interest rates. As a result, this model is less likely to remain appropriate in scenarios of significantly higher SA interest rates than those within the dataset used to calibrate the models.
- Equation 5 assumes a constant elasticity between the Lesotho and SA ZCYs. Specifically, according to Equation 5 (using the Nelder-Mead method), a 1% change in the SA interest rate will lead to a 0.747% change in the Lesotho interest rate. Again, the intuition here is less clear, as we are talking about a percentage change in a percentage figure. It is much clearer to directly relate the level of Lesotho's ZCYs to those of SA.

Table 2: Alternative equations investigated*

Preferred^ $L_i = a + b.SA_i + c.ln(term_i) + \epsilon_i$

1. $L_i = a + b.SA_i + c.term_i + d.ln(term_i) + \epsilon_i$

2. $L_i = a + b.SA_i + c.term_i + \epsilon_i$

3. $L_i = a + b.SA_i + c.SA_i^2 + d.term_i + e.term_i^2 + f.term_i^3 + \epsilon_i$

4. $L_i = a + b.ln(SA_i) + c.term_i + d.ln(term_i) + \epsilon_i$

5. $ln(L_i) = a + b.ln(SA_i) + c.ln(term_i) + \epsilon_i$

* Notation is as defined in Section 0 of this report.

^ The "preferred" equation is the same as that proposed in Section 0.

Table 3: Test results for bond yield estimation

			In-sample			Out-of-sample		
Bootstrapping method	**Terms included**	**Equation^**	**Bias (%)**	**MAE (%)**	**RMSE (%)**	**Bias (%)**	**MAE (%)**	**RMSE (%)**
Nelder-Mead	All	Preferred+	0.01	0.37	0.47	0.01	0.39	0.48
Nelder-Mead	All	1	-0.02	0.37	0.46	-0.01	0.40	0.48
Nelder-Mead	All	2	-0.14	0.45	0.54	-0.13	0.46	0.55
Nelder-Mead	All	3	0.05	0.33	0.42	0.04	0.40	0.47
Nelder-Mead	All	4	-0.01	0.35	0.44	-0.01	0.39	0.46
Nelder-Mead	All	5	-0.02	0.38	0.49	-0.02	0.39	0.48
Nelder-Mead	>1 year*	2	0.12	0.37	0.43	0.14	0.40	0.47
Linear spline	All	Preferred	0.02	0.37	0.48	0.03	0.40	0.49
Linear spline	All	1	-0.19	0.39	0.49	-0.18	0.42	0.51
Linear spline	All	2	-0.33	0.48	0.60	-0.32	0.49	0.61
Linear spline	All	3	-0.05	0.34	0.42	-0.06	0.40	0.48
Linear spline	All	4	-0.16	0.38	0.47	-0.16	0.41	0.50
Linear spline	All	5	-0.22	0.44	0.52	-0.22	0.45	0.53
Linear spline	>1 year*	2	0.02	0.35	0.42	0.03	0.39	0.46

Source: CBL

* For these versions of the model, the data set included only ZCYs corresponding to terms longer than one year, leaving a total of 36 observations instead of the original 112.

^ Equations are set out in Table 2. Estimation output for these models is included in Appendix II.

+ The recommendation of this paper is to use the "preferred" equation combined with the Nelder-Mead bootstrapping method.

MAE = mean absolute error, RMSE = root mean squared error

Other equations listed above are less preferred, based on the following reasons.

- Out of Nelder-Mead and linear spline bootstrapping methods, the Nelder-Mead method leads to better performance under most specifications, leading to less bias and lower RMSE and MAE statistics.
- Equation 2 appears to perform well when only fitted to ZCYs corresponding to terms longer than one year. However, the coefficient of SA_i in these cases falls to an unreasonably low value – close to 0.45 for both bootstrapping methods. This model is therefore likely to become inappropriate in scenarios of significantly higher SA interest rates than those within the dataset used to calibrate the models. The use of only longer-term ZCYs appeared to lead to no available benefit from including non-linear regressors in the equation, which is why only Equation 2 was tested in this way.
- When using the entire dataset, Equation 2 is dominated by the preferred equation, indicating the need to account for non-linearity of interest rates in payment term. This is corroborated by regression diagnostics for Equation 2, which indicate non-normality of errors and a less favourable SIC statistic relative to the preferred equation.
- Equation 3 appears to perform roughly on par with the preferred equation based on the above statistics, perhaps a little better when using the linear spline method of constructing the ZCY curve. However, the use of polynomial terms to capture nonlinearity is highly likely to lead to unreasonable results when SA interest rates or the payment term are beyond the values present in the data used to calibrate the model. Further, Equation 3 leads to non-normality of errors when using the Nelder-Mead method to construct Lesotho's ZCY curve.

(c) Model Limitations

In evaluating this model, its limitations need to be borne in mind in order to achieve maximal benefit from its use. An important limitation of this model is that it is not sensitive to changes in the relationship between Lesotho and SA yields. For example, if there were a change in the perceived level of risk inherent in Lesotho's government securities relative to those of SA, such as might result from a sustained increase in political risk in Lesotho, this change would not be reflected by the model as currently estimated. In such a case the model could be reviewed and re-estimated using up to date information in order to maximise its predictive power. However, this limitation is also a positive feature of the model, since the estimated yield curves are less susceptible to undue influence by individual transactions that are large enough to move Lesotho's bond market.

The model assumes that the fundamental mechanisms facilitating bond markets in SA and Lesotho remain the same as those prevailing during the period from which data is taken. This assumption could be invalidated by, for example, improvement in Lesotho government bond liquidity, such as may arise from capital market development efforts. In the case of such developments, the model's estimates are likely to become less accurate. However, enhanced liquidity would also provide much valuable information about bond yields, which would potentially enable a different modelling approach to be taken.

The model's estimates of yields for terms above 10 years should be considered unreliable, since no data for such yields in Lesotho is available. Subjectively, the authors believe that the model's extrapolations for terms up to 12 years may still be somewhat useful, albeit less reliable. However, estimates by the model for terms above approximately 15 years are unlikely to contain any useful information, as the behaviour of that part of the zero coupon yield curve relative to SA yields is unknown, and could be significantly different to that prevailing at 10 years.

Finally, as mentioned earlier, a limitation of the model is that, being designed as an estimation model, it is less appropriate for isolating and understanding the underlying drivers of Lesotho's ZCY curve, a task which is outside the scope of this project.

These limitations, while important to bear in mind, do not compromise the model's overall contribution. Specifically, as per the test results in Section 0, the model succeeds in providing a reasonably indicative estimation of Lesotho's ZCY curve, in line with its purpose.

6. CONCLUSION

The purpose of this project has been to facilitate capital market development goals by providing estimates of Lesotho's government yield curve to the public. This is expected to improve overall capital allocation in Lesotho by enabling better pricing of financial products, helping encourage corporate bond issuance, and providing guidance as to yields that will be achieved at government bond auctions.

CBL's work to date on yield curve estimation has been based on yield curve smoothing models such as Nelson-Siegel. However, the bond markets to which these techniques are typically applied are far more liquid than Lesotho's bond market, where little to no observable secondary trading takes place. Therefore, a slightly more novel and innovative approach is taken in this paper.

Similar to a methodology that has been used to arrive at a Danish yield curve for regulatory purposes, this paper primarily draws upon interest rate information from SA financial markets, creating a model that is able to estimate Lesotho's ZCY curve using that of SA as an input. The model was calibrated using the scant interest rate data points available from Lesotho government bond issuances and re-openings since 2010, and using bootstrapped ZCYs as the dependent variable to be estimated.

Tests of the estimates produced by the model confirm its suitability, with the preferred model showing very little overall bias in estimates of government bond yields, and featuring MAEs and RMSEs of less than 0.5% based on both in-sample and out-of-sample testing. This suggests that the model, when used to estimate the yield obtainable on a prospective government bond issuance, can be expected to produce estimates that are on average less than 0.5% away from the actual bond yield.

This paper recommends that a) the model be introduced and explained to commercial banks and insurers in Lesotho, b) this paper be published on the CBL website, c) the model's current estimate of Lesotho's government bond yield curve be published daily on the CBL website, and d) the model be re-calibrated every year as new data becomes available.

While this model is a definite improvement on more naïve approaches (such as substituting the SA ZCY for the Lesotho ZCY), the estimates produced are far from perfect. This is inevitable due to data limitations arising from the illiquidity of Lesotho's bond market. Thus the model should be used with appropriate caution and care, bearing in mind its limitations.

7. RECOMMENDATIONS

This paper recommends the following.

- CBL should introduce and explain this work to commercial banks and insurers within Lesotho
- This can be done at the technical level through the Financial Markets Forum, and should also be done at the executive level
- The final version of this paper should be published on the CBL website, to provide transparency regarding the process used to derive CBL's ZCY estimates
- An estimated yield curve should be published daily on the CBL website
- Estimates should be provided from 2 years of maturity going up to 10 years in increments of one year. For 1 year of maturity, the yield obtained at the most recent auction of 364-day treasury bills is likely to be a more reliable estimator, and could be shown alongside the model's estimates for longer-term yields.
- A spreadsheet should be created to produce the estimates daily, taking SA market data as an input and applying the model developed
- The estimates should be clearly labelled as estimates, to avoid confusion
- Every year, the model should be recalibrated (i.e. the parameters should be re-estimated) if there have been new bond issuances or re-openings since the last recalibration
- A spreadsheet to automate this, or detailed guidance on how to carry this out, should be created

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Appendix I: Estimated zero-coupon yields

Yields in this Appendix are expressed as continuously compounded annual interest rates.

Date (dd/mm/yyyy)	Approximate term (years)	SA yield	Lesotho yield, based on Nelder-Mead	Lesotho yield, based on linear spline
20/10/2010	0.25	5.99%	5.95%	5.94%
20/10/2010	0.5	5.86%	6.09%	6.09%
20/10/2010	0.75	5.80%	6.87%	6.87%
20/10/2010	1	5.79%	7.05%	7.05%
20/10/2010	3	6.36%	8.16%	8.17%
20/10/2010	5	6.96%	8.96%	8.97%
08/12/2010	0.25	5.55%	5.70%	5.70%
08/12/2010	0.5	5.50%	6.02%	6.00%
08/12/2010	0.75	5.49%	6.79%	6.78%
08/12/2010	1	5.52%	6.94%	6.94%
08/12/2010	2.75	6.25%	8.58%	8.57%
08/12/2010	4.75	7.09%	9.20%	9.22%
16/02/2011	0.25	5.58%	5.48%	5.47%
16/02/2011	0.5	5.62%	5.76%	5.76%
16/02/2011	0.75	5.69%	6.52%	6.53%
16/02/2011	1	5.83%	6.68%	6.68%
16/02/2011	2.75	7.03%	7.92%	7.92%
16/02/2011	4.75	7.86%	8.51%	8.52%
20/04/2011	0.25	5.58%	5.39%	5.39%
20/04/2011	0.5	5.61%	5.61%	5.61%
20/04/2011	0.75	5.73%	6.04%	6.04%
20/04/2011	1	5.88%	6.49%	6.49%
20/04/2011	2.5	6.97%	8.19%	8.20%
20/04/2011	4.5	7.76%	8.94%	8.96%
22/06/2011	0.25	5.58%	5.33%	5.33%
22/06/2011	0.5	5.65%	5.56%	5.56%
22/06/2011	0.75	5.78%	5.99%	5.99%
22/06/2011	1	5.93%	6.44%	6.44%
22/06/2011	4.25	7.48%	9.21%	9.23%
22/06/2011	10	8.25%	10.00%	10.13%
17/08/2011	0.25	5.60%	5.33%	5.32%
17/08/2011	0.5	5.58%	5.42%	5.42%
17/08/2011	0.75	5.58%	5.88%	5.89%
17/08/2011	1	5.59%	6.20%	6.20%
17/08/2011	4.25	6.62%	8.82%	8.84%
17/08/2011	9.75	7.76%	10.37%	10.47%

Date (dd/mm/yyyy)	Approximate term (years)	SA yield	Lesotho yield, based on Nelder-Mead	Lesotho yield, based on linear spline
19/10/2011	0.25	5.58%	5.32%	5.32%
19/10/2011	0.5	5.51%	5.41%	5.41%
19/10/2011	0.75	5.45%	5.82%	5.82%
19/10/2011	1	5.43%	6.10%	6.10%
19/10/2011	4	6.52%	9.04%	9.04%
19/10/2011	9.75	7.99%	9.85%	10.02%
21/12/2011	0.25	5.59%	5.31%	5.31%
21/12/2011	0.5	5.58%	5.34%	5.34%
21/12/2011	0.75	5.58%	5.68%	5.68%
21/12/2011	1	5.60%	5.95%	5.95%
21/12/2011	3.75	6.61%	9.17%	9.17%
21/12/2011	9.5	7.97%	9.91%	10.13%
15/02/2012	0.25	5.60%	5.47%	5.46%
15/02/2012	0.5	5.61%	5.47%	5.46%
15/02/2012	0.75	5.63%	5.68%	5.68%
15/02/2012	1	5.67%	5.94%	5.94%
15/02/2012	7	7.30%	8.59%	8.63%
15/02/2012	9.25	7.67%	11.06%	11.06%
18/04/2012	0.25	5.60%	5.58%	5.56%
18/04/2012	0.5	5.61%	5.62%	5.62%
18/04/2012	0.75	5.65%	5.87%	5.87%
18/04/2012	1	5.70%	6.02%	6.02%
18/04/2012	6.75	7.28%	8.74%	8.75%
18/04/2012	9.25	7.67%	10.52%	10.51%
15/08/2012	0.25	5.08%	5.46%	5.46%
15/08/2012	0.5	5.02%	5.54%	5.54%
15/08/2012	0.75	4.99%	5.77%	5.77%
15/08/2012	1	4.98%	5.66%	5.66%
15/08/2012	6.5	6.42%	8.69%	8.64%
15/08/2012	8.75	6.88%	9.12%	9.10%
17/10/2012	0.25	5.08%	5.47%	5.45%
17/10/2012	0.5	4.96%	5.40%	5.41%
17/10/2012	0.75	4.89%	5.72%	5.72%
17/10/2012	1	4.86%	5.76%	5.77%
17/10/2012	6.25	6.29%	8.79%	8.77%
17/10/2012	8.75	6.93%	8.65%	8.67%
19/12/2012	0.25	5.13%	5.38%	5.37%
19/12/2012	0.5	5.06%	5.36%	5.37%
19/12/2012	0.75	5.02%	5.63%	5.63%

Date (dd/mm/yyyy)	Approximate term (years)	SA yield	Lesotho yield, based on Nelder-Mead	Lesotho yield, based on linear spline
19/12/2012	1	4.99%	5.69%	5.69%
19/12/2012	6.25	6.16%	8.46%	8.44%
19/12/2012	8.5	6.67%	8.92%	8.92%
20/02/2013	0.25	5.08%	5.37%	5.36%
20/02/2013	0.5	5.08%	5.34%	5.34%
20/02/2013	0.75	5.06%	5.54%	5.53%
20/02/2013	1	5.06%	5.54%	5.54%
20/02/2013	6	6.30%	8.30%	8.27%
20/02/2013	8.25	6.82%	9.24%	9.23%
19/03/2014	0.25	5.73%	6.05%	6.05%
19/03/2014	0.5	5.95%	6.30%	6.30%
19/03/2014	0.75	6.15%	6.60%	6.60%
19/03/2014	1	6.38%	6.68%	6.67%
19/03/2014	5	8.07%	8.28%	8.28%
25/06/2014	0.25	5.81%	6.15%	6.15%
25/06/2014	0.5	6.02%	6.35%	6.35%
25/06/2014	0.75	6.17%	6.68%	6.68%
25/06/2014	1	6.31%	6.70%	6.70%
25/06/2014	4.75	7.51%	8.22%	8.21%
17/08/2014	0.25	6.08%	6.16%	6.15%
17/08/2014	0.5	6.15%	6.34%	6.32%
17/08/2014	0.75	6.25%	6.63%	6.68%
17/08/2014	1	6.34%	6.99%	6.95%
17/08/2014	4.5	7.37%	8.40%	8.43%
17/08/2014	6.75	7.83%	10.55%	10.54%
31/12/2014	0.25	6.13%	6.30%	6.30%
31/12/2014	0.5	6.26%	6.88%	6.88%
31/12/2014	0.75	6.37%	7.02%	7.02%
31/12/2014	1	6.48%	7.14%	7.14%
31/12/2014	4	7.31%	8.74%	8.74%
31/12/2014	6.5	7.69%	10.08%	10.08%
18/03/2015	0.25	6.11%	6.33%	6.30%
18/03/2015	0.5	6.19%	6.66%	6.67%
18/03/2015	0.75	6.27%	7.00%	6.86%
18/03/2015	1	6.38%	6.97%	7.09%
18/03/2015	4	7.25%	8.22%	8.19%
18/03/2015	6.25	7.59%	10.33%	10.29%

Source: CBL, Bloomberg

Appendix II: Model estimation output for alternative models

ZCY curve-fitting method	**Terms included**	**Equation^**	**Estimation output (Eviews 8 statistical package used)**
Nelder-Mead	All	Preferred	Dependent Variable: LESOTHO_YIELD Method: Least Squares Date: 10/14/15 Time: 09:37 Sample: 1 112 Included observations: 112 White heteroskedasticity-consistent standard errors & covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.015959	0.005300	3.011016	0.0032
SA_YIELD	0.887014	0.092088	9.632233	0.0000
LOG(TERM)	0.007336	0.000489	15.00939	0.0000

R-squared	0.922860	Mean dependent var	0.069946
Adjusted R-squared	0.921445	S.D. dependent var	0.015749
S.E. of regression	0.004414	Akaike info criterion	-7.981620
Sum squared resid	0.002124	Schwarz criterion	-7.908803
Log likelihood	449.9707	Hannan-Quinn criter.	-7.952076
F-statistic	652.0080	Durbin-Watson stat	1.646658
Prob(F-statistic)	0.000000	Wald F-statistic	451.0242
Prob(Wald F-statistic)	0.000000		

Jarque-Bera normality test p-value: 0.591
Breusch-Pagan-Godfrey heteroskedasticity test p-value: 0.000

ZCY curve-fitting method	Terms included	Equation^	Estimation output (Eviews 8 statistical package used)
Nelder-Mead	All	1	Dependent Variable: LESOTHO_YIELD Method: Least Squares Date: 07/30/15 Time: 15:15 Sample: 1 112 Included observations: 112 White heteroskedasticity-consistent standard errors & covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.018226	0.005039	3.616994	0.0005
SA_YIELD	0.809073	0.091084	8.882755	0.0000
TERM	0.001138	0.000430	2.644595	0.0094
LOG(TERM)	0.005229	0.000914	5.720503	0.0000

R-squared	0.928525	Mean dependent var	0.069946
Adjusted R-squared	0.926540	S.D. dependent var	0.015749
S.E. of regression	0.004269	Akaike info criterion	-8.040038
Sum squared resid	0.001968	Schwarz criterion	-7.942948
Log likelihood	454.2421	Hannan-Quinn criter.	-8.000646
F-statistic	467.6730	Durbin-Watson stat	1.680148
Prob(F-statistic)	0.000000	Wald F-statistic	323.9629
Prob(Wald F-statistic)	0.000000		

Jarque-Bera normality test p-value: 0.456
Breusch-Pagan-Godfrey heteroskedasticity test p-value: 0.000

ZCY curve-fitting method	**Terms included**	**Equation^**	**Estimation output (Eviews 8 statistical package used)**
Nelder-Mead	All	2	Dependent Variable: LESOTHO_YIELD Method: Least Squares Date: 07/30/15 Time: 15:13 Sample: 1 112 Included observations: 112 White heteroskedasticity-consistent standard errors & covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.008059	0.004851	1.661358	0.0995
SA_YIELD	0.923599	0.089494	10.32025	0.0000
TERM	0.002885	0.000252	11.46664	0.0000

R-squared	0.906938	Mean dependent var	0.069946
Adjusted R-squared	0.905230	S.D. dependent var	0.015749
S.E. of regression	0.004848	Akaike info criterion	-7.793974
Sum squared resid	0.002562	Schwarz criterion	-7.721157
Log likelihood	439.4625	Hannan-Quinn criter.	-7.764430
F-statistic	531.1300	Durbin-Watson stat	1.541317
Prob(F-statistic)	0.000000	Wald F-statistic	339.4071
Prob(Wald F-statistic)	0.000000		

Jarque-Bera normality test p-value: 0.059
Breusch-Pagan-Godfrey heteroskedasticity test p-value: 0.000

ZCY curve-fitting method	Terms included	Equation^	Estimation output (Eviews 8 statistical package used)
Nelder-Mead	All	3	See estimation output below

Dependent Variable: LESOTHO_YIELD
Method: Least Squares
Date: 07/30/15 Time: 15:14
Sample: 1 112
Included observations: 112
White heteroskedasticity-consistent standard errors & covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.048169	0.026418	-1.823326	0.0711
SA_YIELD	2.811290	0.913532	3.077385	0.0027
SA_YIELD^2	-17.24530	8.046874	-2.143106	0.0344
TERM	0.010931	0.002035	5.372876	0.0000
TERM^2	-0.001588	0.000538	-2.952347	0.0039
TERM^3	8.65E-05	3.79E-05	2.282722	0.0244

R-squared	0.935896	Mean dependent var	0.069946
Adjusted R-squared	0.932872	S.D. dependent var	0.015749
S.E. of regression	0.004080	Akaike info criterion	-8.113164
Sum squared resid	0.001765	Schwarz criterion	-7.967530
Log likelihood	460.3372	Hannan-Quinn criter.	-8.054075
F-statistic	309.5126	Durbin-Watson stat	1.614106
Prob(F-statistic)	0.000000	Wald F-statistic	264.8967
Prob(Wald F-statistic)	0.000000		

Jarque-Bera normality test p-value: 0.045
Breusch-Pagan-Godfrey heteroskedasticity test p-value: 0.000

ZCY curve-fitting method	**Terms included**	**Equation^**	**Estimation output (Eviews 8 statistical package used)**
Nelder-Mead	All	4	Dependent Variable: LESOTHO_YIELD Method: Least Squares Date: 07/30/15 Time: 15:16 Sample: 1 112 Included observations: 112 White heteroskedasticity-consistent standard errors & covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.205139	0.013595	15.08905	0.0000
LOG(SA_YIELD)	0.049104	0.004644	10.57385	0.0000
TERM	0.001237	0.000417	2.968268	0.0037
LOG(TERM)	0.005120	0.000875	5.853085	0.0000

R-squared	0.930924	Mean dependent var	0.069946
Adjusted R-squared	0.929005	S.D. dependent var	0.015749
S.E. of regression	0.004196	Akaike info criterion	-8.074180
Sum squared resid	0.001902	Schwarz criterion	-7.977090
Log likelihood	456.1541	Hannan-Quinn criter.	-8.034787
F-statistic	485.1663	Durbin-Watson stat	1.653306
Prob(F-statistic)	0.000000	Wald F-statistic	328.4225
Prob(Wald F-statistic)	0.000000		

Jarque-Bera normality test p-value: 0.358
Breusch-Pagan-Godfrey heteroskedasticity test p-value: 0.000

<table>
<tr><th>ZCY curve-fitting method</th><th>Terms included</th><th>Equation^</th><th>Estimation output (Eviews 8 statistical package used)</th></tr>
<tr><td>Nelder-Mead</td><td>All</td><td>5</td><td>
Dependent Variable: LOG(LESOTHO_YIELD)
Method: Least Squares
Date: 07/30/15 Time: 15:18
Sample: 1 112
Included observations: 112
White heteroskedasticity-consistent standard errors & covariance
<table>
<tr><th>Variable</th><th>Coefficient</th><th>Std. Error</th><th>t-Statistic</th><th>Prob.</th></tr>
<tr><td>C</td><td>-0.584628</td><td>0.180581</td><td>-3.237483</td><td>0.0016</td></tr>
<tr><td>LOG(SA_YIELD)</td><td>0.747236</td><td>0.063194</td><td>11.82458</td><td>0.0000</td></tr>
<tr><td>LOG(TERM)</td><td>0.099237</td><td>0.006325</td><td>15.69082</td><td>0.0000</td></tr>
</table>
<table>
<tr><td>R-squared</td><td>0.929820</td><td>Mean dependent var</td><td>-2.683329</td></tr>
<tr><td>Adjusted R-squared</td><td>0.928532</td><td>S.D. dependent var</td><td>0.212930</td></tr>
<tr><td>S.E. of regression</td><td>0.056924</td><td>Akaike info criterion</td><td>-2.867795</td></tr>
<tr><td>Sum squared resid</td><td>0.353192</td><td>Schwarz criterion</td><td>-2.794978</td></tr>
<tr><td>Log likelihood</td><td>163.5965</td><td>Hannan-Quinn criter.</td><td>-2.838250</td></tr>
<tr><td>F-statistic</td><td>722.0715</td><td>Durbin-Watson stat</td><td>1.399653</td></tr>
<tr><td>Prob(F-statistic)</td><td>0.000000</td><td>Wald F-statistic</td><td>619.2950</td></tr>
<tr><td>Prob(Wald F-statistic)</td><td>0.000000</td><td></td><td></td></tr>
</table>
Jarque-Bera normality test p-value: 0.635
Breusch-Pagan-Godfrey heteroskedasticity test p-value: 0.025
</td></tr>
</table>

ZCY curve-fitting method	Terms included	Equation^	Estimation output (Eviews 8 statistical package used)
Nelder-Mead	>1 year*	2	Dependent Variable: LESOTHO_YIELD Method: Least Squares Date: 07/30/15 Time: 15:18 Sample: 1 36 Included observations: 36

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.048719	0.012430	3.919504	0.0004
SA_YIELD	0.439234	0.190448	2.306320	0.0275
TERM	0.001909	0.000463	4.125539	0.0002

R-squared	0.513801	Mean dependent var	0.090757
Adjusted R-squared	0.484335	S.D. dependent var	0.008051
S.E. of regression	0.005781	Akaike info criterion	-7.388787
Sum squared resid	0.001103	Schwarz criterion	-7.256827
Log likelihood	135.9982	Hannan-Quinn criter.	-7.342729
F-statistic	17.43675	Durbin-Watson stat	2.143940
Prob(F-statistic)	0.000007		

Jarque-Bera normality test p-value: 0.412
Breusch-Pagan-Godfrey heteroskedasticity test p-value: 0.143

ZCY curve-fitting method	Terms included	Equation^	Estimation output (Eviews 8 statistical package used)
Linear spline	All	Preferred	Dependent Variable: LESOTHO_YIELD Method: Least Squares Date: 10/14/15 Time: 10:15 Sample: 1 112 Included observations: 112 White heteroskedasticity-consistent standard errors & covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.015057	0.005298	2.841820	0.0054
SA_YIELD	0.902939	0.092113	9.802520	0.0000
LOG(TERM)	0.007334	0.000482	15.20621	0.0000

R-squared	0.925150	Mean dependent var	0.069986
Adjusted R-squared	0.923777	S.D. dependent var	0.015856
S.E. of regression	0.004378	Akaike info criterion	-7.998210
Sum squared resid	0.002089	Schwarz criterion	-7.925393
Log likelihood	450.8997	Hannan-Quinn criter.	-7.968665
F-statistic	673.6226	Durbin-Watson stat	1.624196
Prob(F-statistic)	0.000000	Wald F-statistic	473.2138
Prob(Wald F-statistic)	0.000000		

Jarque-Bera normality test p-value: 0.508
Breusch-Pagan-Godfrey heteroskedasticity test p-value: 0.000

<table>
<tr><th>ZCY curve-fitting method</th><th>Terms included</th><th>Equation^</th><th>Estimation output (Eviews 8 statistical package used)</th></tr>
<tr><td>Linear spline</td><td>All</td><td>1</td><td>Dependent Variable: LESOTHO_YIELD
Method: Least Squares
Date: 07/28/15 Time: 15:11
Sample: 1 112
Included observations: 112
White heteroskedasticity-consistent standard errors & covariance
<table>
<tr><th>Variable</th><th>Coefficient</th><th>Std. Error</th><th>t-Statistic</th><th>Prob.</th></tr>
<tr><td>C</td><td>0.019053</td><td>0.004908</td><td>3.882154</td><td>0.0002</td></tr>
<tr><td>SA_YIELD</td><td>0.769126</td><td>0.086777</td><td>8.863219</td><td>0.0000</td></tr>
<tr><td>TERM</td><td>0.001224</td><td>0.000419</td><td>2.922906</td><td>0.0042</td></tr>
<tr><td>LOG(TERM)</td><td>0.005068</td><td>0.000892</td><td>5.684141</td><td>0.0000</td></tr>
</table>
<table>
<tr><td>R-squared</td><td>0.931491</td><td>Mean dependent var</td><td>0.069986</td></tr>
<tr><td>Adjusted R-squared</td><td>0.929588</td><td>S.D. dependent var</td><td>0.015856</td></tr>
<tr><td>S.E. of regression</td><td>0.004207</td><td>Akaike info criterion</td><td>-8.068878</td></tr>
<tr><td>Sum squared resid</td><td>0.001912</td><td>Schwarz criterion</td><td>-7.971789</td></tr>
<tr><td>Log likelihood</td><td>455.8572</td><td>Hannan-Quinn criter.</td><td>-8.029486</td></tr>
<tr><td>F-statistic</td><td>489.4807</td><td>Durbin-Watson stat</td><td>1.652890</td></tr>
<tr><td>Prob(F-statistic)</td><td>0.000000</td><td>Wald F-statistic</td><td>352.3139</td></tr>
<tr><td>Prob(Wald F-statistic)</td><td>0.000000</td><td></td><td></td></tr>
</table>
Jarque-Bera normality test p-value: 0.195
Breusch-Pagan-Godfrey heteroskedasticity test p-value: 0.000</td></tr>
</table>

ZCY curve-fitting method	**Terms included**	**Equation^**	**Estimation output (Eviews 8 statistical package used)**
Linear spline	All	2	Dependent Variable: LESOTHO_YIELD Method: Least Squares Date: 07/28/15 Time: 12:39 Sample: 1 112 Included observations: 112 White heteroskedasticity-consistent standard errors & covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.009368	0.004768	1.964817	0.0520
SA_YIELD	0.874138	0.085714	10.19828	0.0000
TERM	0.002915	0.000247	11.80795	0.0000

R-squared	0.911499	Mean dependent var	0.069986
Adjusted R-squared	0.909875	S.D. dependent var	0.015856
S.E. of regression	0.004760	Akaike info criterion	-7.830678
Sum squared resid	0.002470	Schwarz criterion	-7.757861
Log likelihood	441.5179	Hannan-Quinn criter.	-7.801133
F-statistic	561.3093	Durbin-Watson stat	1.512213
Prob(F-statistic)	0.000000	Wald F-statistic	389.9149
Prob(Wald F-statistic)	0.000000		

Jarque-Bera normality test p-value: 0.009
Breusch-Pagan-Godfrey heteroskedasticity test p-value: 0.005

ZCY curve-fitting method	**Terms included**	**Equation^**	**Estimation output (Eviews 8 statistical package used)**
Linear spline	All	3	(see below)

Dependent Variable: LESOTHO_YIELD
Method: Least Squares
Date: 07/28/15 Time: 16:13
Sample: 1 112
Included observations: 112
White heteroskedasticity-consistent standard errors & covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.041933	0.024859	-1.686842	0.0946
SA_YIELD	2.531632	0.833897	3.035905	0.0030
SA_YIELD^2	-14.72022	7.130913	-2.064283	0.0414
TERM	0.011173	0.002016	5.541402	0.0000
TERM^2	-0.001685	0.000531	-3.171696	0.0020
TERM^3	9.47E-05	3.73E-05	2.536721	0.0126

R-squared	0.938290	Mean dependent var	0.069986
Adjusted R-squared	0.935379	S.D. dependent var	0.015856
S.E. of regression	0.004031	Akaike info criterion	-8.137680
Sum squared resid	0.001722	Schwarz criterion	-7.992046
Log likelihood	461.7101	Hannan-Quinn criter.	-8.078592
F-statistic	322.3428	Durbin-Watson stat	1.605372
Prob(F-statistic)	0.000000	Wald F-statistic	276.7245
Prob(Wald F-statistic)	0.000000		

Jarque-Bera normality test p-value: 0.266
Breusch-Pagan-Godfrey heteroskedasticity test p-value: 0.000

ZCY curve-fitting method	Terms included	Equation^	Estimation output (Eviews 8 statistical package used)
Linear spline	All	4	Dependent Variable: LESOTHO_YIELD Method: Least Squares Date: 07/28/15 Time: 15:42 Sample: 1 112 Included observations: 112 White heteroskedasticity-consistent standard errors & covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.200691	0.013209	15.19302	0.0000
LOG(SA_YIELD)	0.048093	0.004548	10.57544	0.0000
TERM	0.001330	0.000405	3.283091	0.0014
LOG(TERM)	0.004952	0.000854	5.797435	0.0000

R-squared	0.933699	Mean dependent var	0.069986
Adjusted R-squared	0.931857	S.D. dependent var	0.015856
S.E. of regression	0.004139	Akaike info criterion	-8.101633
Sum squared resid	0.001850	Schwarz criterion	-8.004544
Log likelihood	457.6914	Hannan-Quinn criter.	-8.062241
F-statistic	506.9777	Durbin-Watson stat	1.625074
Prob(F-statistic)	0.000000	Wald F-statistic	353.1348
Prob(Wald F-statistic)	0.000000		

Jarque-Bera normality test p-value: 0.273
Breusch-Pagan-Godfrey heteroskedasticity test p-value: 0.000

<table>
<tr><th>ZCY curve-fitting method</th><th>Terms included</th><th>Equation^</th><th>Estimation output (Eviews 8 statistical package used)</th></tr>
<tr><td>Linear spline</td><td>All</td><td>5</td><td>Dependent Variable: LOG(LESOTHO_YIELD)
Method: Least Squares
Date: 07/28/15 Time: 15:40
Sample: 1 112
Included observations: 112
White heteroskedasticity-consistent standard errors & covariance
<table>
<tr><th>Variable</th><th>Coefficient</th><th>Std. Error</th><th>t-Statistic</th><th>Prob.</th></tr>
<tr><td>C</td><td>-0.647983</td><td>0.173395</td><td>-3.737033</td><td>0.0003</td></tr>
<tr><td>LOG(SA_YIELD)</td><td>0.732457</td><td>0.061297</td><td>11.94927</td><td>0.0000</td></tr>
<tr><td>LOG(TERM)</td><td>0.099416</td><td>0.006252</td><td>15.90194</td><td>0.0000</td></tr>
</table>
<table>
<tr><td>R-squared</td><td>0.931835</td><td>Mean dependent var</td><td>-2.683008</td></tr>
<tr><td>Adjusted R-squared</td><td>0.930585</td><td>S.D. dependent var</td><td>0.213992</td></tr>
<tr><td>S.E. of regression</td><td>0.056380</td><td>Akaike info criterion</td><td>-2.886984</td></tr>
<tr><td>Sum squared resid</td><td>0.346479</td><td>Schwarz criterion</td><td>-2.814167</td></tr>
<tr><td>Log likelihood</td><td>164.6711</td><td>Hannan-Quinn criter.</td><td>-2.857439</td></tr>
<tr><td>F-statistic</td><td>745.0342</td><td>Durbin-Watson stat</td><td>1.389331</td></tr>
<tr><td>Prob(F-statistic)</td><td>0.000000</td><td>Wald F-statistic</td><td>651.0000</td></tr>
<tr><td>Prob(Wald F-statistic)</td><td>0.000000</td><td></td><td></td></tr>
</table>
Jarque-Bera normality test p-value: 0.653
Breusch-Pagan-Godfrey heteroskedasticity test p-value: 0.063</td></tr>
<tr><td>Linear spline</td><td>>1 year*</td><td>2</td><td>Dependent Variable: LESOTHO_YIELD
Method: Least Squares
Date: 07/28/15 Time: 12:41
Sample: 1 36
Included observations: 36
<table>
<tr><th>Variable</th><th>Coefficient</th><th>Std. Error</th><th>t-Statistic</th><th>Prob.</th></tr>
<tr><td>C</td><td>0.046823</td><td>0.011889</td><td>3.938252</td><td>0.0004</td></tr>
<tr><td>SA_YIELD</td><td>0.446054</td><td>0.176117</td><td>2.532709</td><td>0.0163</td></tr>
<tr><td>TERM</td><td>0.001989</td><td>0.000459</td><td>4.336454</td><td>0.0001</td></tr>
</table>
<table>
<tr><td>R-squared</td><td>0.545582</td><td>Mean dependent var</td><td>0.090919</td></tr>
<tr><td>Adjusted R-squared</td><td>0.518042</td><td>S.D. dependent var</td><td>0.008249</td></tr>
<tr><td>S.E. of regression</td><td>0.005727</td><td>Akaike info criterion</td><td>-7.407607</td></tr>
<tr><td>Sum squared resid</td><td>0.001082</td><td>Schwarz criterion</td><td>-7.275647</td></tr>
<tr><td>Log likelihood</td><td>136.3369</td><td>Hannan-Quinn criter.</td><td>-7.361550</td></tr>
<tr><td>F-statistic</td><td>19.81022</td><td>Durbin-Watson stat</td><td>2.066459</td></tr>
<tr><td>Prob(F-statistic)</td><td>0.000002</td><td></td><td></td></tr>
</table>
Jarque-Bera normality test p-value: 0.460
Breusch-Pagan-Godfrey heteroskedasticity test p-value: 0.280</td></tr>
</table>

Source: CBL.

ZCY curve-fitting method	Terms included	Equation^	Estimation output (Eviews 8 statistical package used)

* For these versions of the model, the data set included only ZCYs corresponding to terms longer than one year, leaving a total of 36 observations instead of the original 112.
^ Equations are set out in Table 3.